

$$\begin{aligned}
I_{n+1} &= x^{n+1}e^x - (n+1)e^x \left(x^n + \sum_{k=1}^n (-1)^k n(n-1)\cdots(n-k+1)x^{n-k} \right) \\
&= e^x \left(x^{n+1} - (n+1)x^n - (n+1) \sum_{k=1}^n (-1)^k n(n-1)\cdots(n-k+1)x^{n-k} \right) \\
&= e^x \left(x^{n+1} - (n+1)x^n - \sum_{k=1}^n (-1)^k (n+1)n(n-1)\cdots(n-k+1)x^{n-k} \right) \\
&= e^x \left(x^{n+1} + \sum_{k=1}^{n+1} (-1)^k (n+1)n(n-1)\cdots(n+1-k+1)x^{n+1-k} \right)
\end{aligned}$$